Journal of Engineering Physics and Thermophysics, Vol. 80, No. 2, 2007

CALCULATION OF TURBULENT MOMENTUM TRANSFER IN A SOLID PHASE BASED ON EQUATIONS FOR THE SECOND AND THIRD MOMENTS OF PARTICLE-VELOCITY PULSATIONS

B. B. Rokhman UDC 532.529:662.62

A chain of transfer equations for the second and third moments of dispersed-phase-velocity pulsations in the anisotropic field of energy of random particle motion is obtained based on the computational procedure developed. The interphase and interparticle interactions are allowed for. The turbulent characteristics of the gas are calculated on the basis of a one-parameter turbulence model generalized to the case of two-phase turbulent flows.

In developing methods for calculation of the aerodynamics of two-phase turbulent flows, one must first formulate the initial system of mass- and momentum-transfer equations for the actual parameters of flow of a heterogeneous medium and thereafter pass to the equations for the averaged quantities using the Reynolds procedure. Unlike the laminar case, the system of averaged turbulent-flow equations is open, since the second moments of the pulsation characteristics of a gasdispersed flow are present here in addition to the mean values of velocity, pressure, density, etc. [1]. Using the same Reynolds procedure, we can construct the equations of transfer of second moments, which contain third moments, etc. Therefore, to obtain a closed system of equations one should "break" this process at a certain step, i.e., introduce additional hypotheses for the relationship between the highest and lowest correlation moments.

In the present work, within the framework of the Eulerian approach, i.e., in the case of the so-called double-fluid models, we have formulated a stationary, isothermal, axisymmetric system of averaged differential equations; the system describes, in the narrow-channel approximation, the ascending motion of a gas suspension on the stabilized portion of a pipe with allowance for the interphase and interparticle interaction and for the influence of the channel wall and mass forces. The pulsation characteristics of the gas are calculated with the one-parameter turbulence model generalized to the case of two-phase turbulent flows. To compute the correlation moment $\langle u'_p v'_p \rangle$ appearing in the averaged equation of particle motion we use the Boussinesq hypothesis, according to which $\langle u'_p v'_p \rangle$ is equal to $-\eta_{t,p}\partial u_p/\partial r$. In such an approach, it becomes necessary to determine the proportionality factor (coefficient of turbulent viscosity of the "gas" of particles $\eta_{t,p}$) dependent on the Reynolds stress $\langle v'_p v'_p \rangle$ [2–4]. In turn, the correlation $\langle v'_p v'_p \rangle$ is a function of the second ($\langle w'_p w'_p \rangle$ and $\langle w'_p v'_p \rangle$) and third ($\langle v'_p v'_p v'_p \rangle$, $\langle v'_p w'_p w'_p \rangle$, $\langle v'_p v'_p w'_p \rangle$, and $\langle w'_p w'_p w'_p \rangle$) moments (see below). To compute them we use a specially developed computational procedure; the procedure is based on construction of the equations of transfer of the correlations sought, in which the pseudoturbulent (due to the interparticle collisions) and turbulent effects are allowed for. Equations for the third moments are closed on the basis of a combined approach which involves two methods of determination of the fourth moments appearing in these equations: the first is based on representation of the fourth moments as the sum of the products of the second moments, whereas the second provides for computing the quantities sought from the equations of transfer of the correlations themselves.

On the portion of stabilized flow of the ascending two-phase flow, the averaged radial motion of the gas and particles is absent ($v_g = 0$ and $v_p = 0$) and the averaged parameters remain constant in the longitudinal direction:

$$\partial u_g/\partial z = \partial k_g/\partial z = \partial u_p/\partial z = \partial \langle w'_p v'_p \rangle/\partial z = \partial \langle v'_p v'_p \rangle/\partial z = \partial \langle w'^2_p \rangle/\partial z = \partial \langle w'^3_p \rangle/\partial z = 0 .$$

Institute of Coal Power Technologies, National Academy of Sciences and Ministry of Fuel and Power of Ukraine, 19 Andreevskaya Str., Kiev, 04070, Ukraine. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 80, No. 2, pp. 60–70, March–April, 2007. Original article submitted June 12, 2005.

1062-0125/07/8002-0273 ©2007 Springer Science+Business Media, Inc.

Furthermore, it is assumed that the true volume concentration of the solid phase is uniformly distributed over the channel cross section. With allowance for what has been said above, the system of transfer of the averaged and pulsation parameters of the gas-dispersed flow may be represented as follows:

$$\frac{\rho_g}{r}\frac{\partial}{\partial r}\left[r\,(\eta_{t,g}+\eta_g)\frac{\partial u_g}{\partial r}\right]-\frac{\partial P}{\partial z}-F_{az}=0\,,\quad \frac{\rho_p\beta}{r}\frac{\partial}{\partial r}\left(r\,\langle u'_p v'_p\rangle\right)-F_{az}+\rho_p\beta g=0\,. \tag{1}$$

The left-hand sides of the phase-momentum equations (1) allow for the viscous and Reynolds stresses, the pressure gradient, and the drag and gravity forces.

To determine the coefficient of turbulent viscosity of the carrier flow we use the one-parameter turbulence model, i.e., this system of equations is supplemented with the equation of transfer of the turbulent gas energy [1]:

$$\frac{\rho_g}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\eta_{t,g}}{\sigma_k}+\eta_g\right)\frac{\partial k_g}{\partial r}\right]+\rho_g\eta_{t,g}\left(\frac{\partial u_g}{\partial r}\right)^2-\rho_g\,(\varepsilon_g+\varepsilon_p)+G=0\,. \tag{2}$$

The first term of Eq. (2) describes the molecular and turbulent transfers of pulsation energy, the second term corresponds to its generation due to the averaged-motion energy, the third and fourth terms describe its dissipation due to the viscosity of the gas and the presence of a solid phase in it, and the last term describes the generation of turbulent energy in the trails behind the particles.

As has been noted above, to compute the coefficient of turbulent viscosity of the particle "gas" $\eta_{t,p}$ we must find second and third correlation moments. For this purpose we construct the equations of transfer of the quantities sought. To derive the equations of transfer of the Reynolds stresses $\langle v'_p v'_p\rangle$, $\langle w'_p w'_p\rangle$, and $\langle w'_p v'_p\rangle$ we must primarily obtain the equations of pulsation motion of particles along the radial and transverse axes. For this purpose, we project an image of the actual equation of motion of the dispersed phase onto the coordinate axes indicated. With account for the axial symmetry of the problem ($\partial/\partial\varphi = 0$), projections of the equations of particle motion have the form

$$\rho_p\beta\left(\hat{u}_p\frac{\partial\hat{v}_p}{\partial z}+\hat{v}_p\frac{\partial\hat{v}_p}{\partial r}-\frac{\hat{w}_p^2}{r}\right)=\hat{F}_{ar}\,,\quad \rho_p\beta\left(\hat{u}_p\frac{\partial\hat{w}_p}{\partial z}+\hat{v}_p\frac{\partial\hat{w}_p}{\partial r}+\frac{\hat{v}_p\hat{w}_p}{r}\right)=\hat{F}_{a\varphi} \tag{3}$$

(it is assumed that $\hat{\beta} = \beta$). Applying the Reynolds procedure ($\hat{w}_p = w_p + w'_p$ and $\hat{v}_p = v_p + v'_p$) to Eqs. (3), we obtain

$$\rho_p\beta\left[u_p\frac{\partial v_p}{\partial z}+u_p\frac{\partial v'_p}{\partial z}+u'_p\frac{\partial v_p}{\partial z}+u'_p\frac{\partial v'_p}{\partial z}+v_p\frac{\partial v_p}{\partial r}+v_p\frac{\partial v'_p}{\partial r}+v'_p\frac{\partial v_p}{\partial r}+v'_p\frac{\partial v'_p}{\partial r}-\right.$$

$$\left.-\frac{1}{r}\left(w_pw_p+w_pw'_p+w_pw'_p+w'_pw'_p\right)\right]=F_{ar}+F'_{ar}\,,$$

$$\rho_p\beta\left[u_p\frac{\partial w_p}{\partial z}+u_p\frac{\partial w'_p}{\partial z}+u'_p\frac{\partial w_p}{\partial z}+u'_p\frac{\partial w'_p}{\partial z}+v_p\frac{\partial w_p}{\partial r}+v_p\frac{\partial w'_p}{\partial r}+v'_p\frac{\partial w_p}{\partial r}+v'_p\frac{\partial w'_p}{\partial r}+\right.$$

$$\left.+\frac{1}{r}\left(v_pw_p+v_pw'_p+w_pv'_p+w'_pv'_p\right)\right]=F_{a\varphi}+F'_{a\varphi}\,. \tag{4}$$

Averaging Eqs. (4) with account for $\langle w'_p\rangle = \langle v'_p\rangle = \langle u'_p\rangle = w_p = w_g = 0$, we write

$$\rho_p\beta\left[u_p\frac{\partial v_p}{\partial z}+\frac{\partial\,\langle u'_pv'_p\rangle}{\partial z}+v_p\frac{\partial v_p}{\partial r}+\frac{\partial\,(r\,\langle v'_pv'_p\rangle)}{r\partial r}-\frac{\langle w'_pw'_p\rangle}{r}\right]=F_{ar}\,,$$

$$\rho_p\beta\left(\frac{\partial\,\langle u'_pw'_p\rangle}{\partial z}+\frac{\partial\,(r\,\langle v'_pw'_p\rangle)}{r\partial r}+\frac{1}{r}\,\langle w'_pv'_p\rangle\right)=0\,. \tag{5}$$

In transforming (5), we use the pulsation continuity equation. In accordance with the coordinate selected (e.g., φ), we first multiply the equation by the pulsation of the projection of the particle-velocity vector onto this axis, i.e., by the quantity $w'_{\rm p}$, and thereafter average it.

Subtracting the averaged equations (5) from the actual equations (4), we obtain the equations for the pulsation velocity of particles along the radial and transverse axes:

$$\rho_{\rm p}\beta\left[u_{\rm p}\frac{\partial v'_{\rm p}}{\partial z}+u'_{\rm p}\frac{\partial v_{\rm p}}{\partial z}+u'_{\rm p}\frac{\partial v'_{\rm p}}{\partial z}+v_{\rm p}\frac{\partial v'_{\rm p}}{\partial r}+v'_{\rm p}\frac{\partial v_{\rm p}}{\partial r}+v'_{\rm p}\frac{\partial v'_{\rm p}}{\partial r}-\frac{1}{r}w'_{\rm p}w'_{\rm p}-\frac{\partial\,\langle u'_{\rm p}v'_{\rm p}\rangle}{\partial z}-\right.$$
$$\left.-\frac{\partial\,(r\,\langle v'_{\rm p}v'_{\rm p}\rangle)}{r\partial r}+\frac{1}{r}\,\langle w'_{\rm p}w'_{\rm p}\rangle\right]=F'_{{\rm a}r}\,, \tag{6}$$

$$\rho_{\rm p}\beta\left[u_{\rm p}\frac{\partial w'_{\rm p}}{\partial z}+u'_{\rm p}\frac{\partial w'_{\rm p}}{\partial z}+v_{\rm p}\frac{\partial w'_{\rm p}}{\partial r}+v'_{\rm p}\frac{\partial w'_{\rm p}}{\partial r}+\frac{1}{r}\left(v_{\rm p}w'_{\rm p}+w'_{\rm p}v'_{\rm p}\right)-\frac{\partial\,\langle u'_{\rm p}w'_{\rm p}\rangle}{\partial z}-\right.$$
$$\left.-\frac{\partial\,(r\,\langle v'_{\rm p}w'_{\rm p}\rangle)}{r\partial r}-\frac{1}{r}\,\langle w'_{\rm p}v'_{\rm p}\rangle\right]=F'_{{\rm a}\varphi}\,, \tag{7}$$

where

$$F'_{{\rm a}r}=\frac{\rho_{\rm p}\beta}{\tau}\left(v'_{\rm g}-v'_{\rm p}\right);\quad F'_{{\rm a}\varphi}=\frac{\rho_{\rm p}\beta}{\tau}\left(w'_{\rm g}-w'_{\rm p}\right). \tag{8}$$

To obtain the equations of transfer of normal Reynolds stresses $\langle v'_{\rm p}v'_{\rm p}\rangle$ and $\langle w'_{\rm p}w'_{\rm p}\rangle$ we must multiply Eq. (6) by the quantity $v'_{\rm p}$ and (7) by $w'_{\rm p}$ and carry out averaging. After simple manipulations with account for (8) in the approximation of a boundary layer on the portion of steady-state motion of the gas suspension the equations of transfer of the moments sought take the form

$$\rho_{\rm p}\beta\left[-\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\langle v'_{\rm p}v'_{\rm p}v'_{\rm p}\rangle\right)+\frac{2}{r}\,\langle v'_{\rm p}w'_{\rm p}w'_{\rm p}\rangle\right]+\frac{2\beta\rho_{\rm p}}{\tau}\left(\langle v'_{\rm g}v'_{\rm p}\rangle-\langle v'^2_{\rm p}\rangle\right)+$$
$$+\,2\left\{\frac{\delta^2\rho_{\rm p}}{6912\beta}\left(\frac{\partial u_{\rm p}}{\partial r}\right)^2\left(\frac{1-K_{\rm n}}{2}-\frac{1-K_\tau}{7}\right)^2-C_1\rho_{\rm p}\beta\,\langle v'^2_{\rm p}\rangle\,(1-K_{\rm n}^2)\right\}N=0\,,$$
$$\rho_{\rm p}\beta\left[-\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\langle v'_{\rm p}w'_{\rm p}w'_{\rm p}\rangle\right)-\frac{2}{r}\,\langle v'_{\rm p}w'_{\rm p}w'_{\rm p}\rangle\right]+\frac{2\beta\rho_{\rm p}}{\tau}\left(\langle w'_{\rm g}w'_{\rm p}\rangle-\langle w'^2_{\rm p}\rangle\right)+ \tag{9}$$
$$+\,2\left\{\frac{\delta^2\rho_{\rm p}}{6912\beta}\left(\frac{\partial u_{\rm p}}{\partial r}\right)^2\left(\frac{1-K_{\rm n}}{2}-\frac{1-K_\tau}{7}\right)^2-C_2\rho_{\rm p}\beta\,\langle w'^2_{\rm p}\rangle\,(1-K_{\rm n}^2)\right\}N=0\,,\quad K_{\rm n}<0\,.$$

Equations (9) involve additional terms of nonturbulent origin (last terms of the equations), which describe the generation and dissipation of the pseudoturbulent energy of the solid phase — the phenomena caused by the collisions between particles due to their averaged and pulsatory motions. As has been noted in [3, 4], these terms cannot be computed by the traditional methods of turbulence theory, since the pulsations due to interparticle collisions are mainly dependent on the random position of a unit vector directed along the line of impact. Therefore, to determine these terms we used a specially developed computational procedure based on an analysis of the dynamics of the process of collisions [3, 4]. We may obtain, in a similar manner, the equation of transfer of the quantity $\langle w'_{\rm p}v'_{\rm p}\rangle$:

$$\rho_{\rm p}\beta\left[-\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\langle v'_{\rm p}v'_{\rm p}w'_{\rm p}\rangle\right)+\frac{1}{r}\,\langle w'_{\rm p}w'_{\rm p}w'_{\rm p}\rangle-\frac{1}{r}\,\langle v'_{\rm p}v'_{\rm p}w'_{\rm p}\rangle\right]+\frac{\beta\rho_{\rm p}}{\tau}\left(\langle v'_{\rm g}w'_{\rm p}\rangle+\langle v'_{\rm p}w'_{\rm g}\rangle-2\,\langle w'_{\rm p}v'_{\rm p}\rangle\right)=0\,. \tag{10}$$

The mixed correlation moments of second order appearing in Eqs. (9) and (10) are determined in terms of the correlations of the carrier flow in a locally homogeneous approximation in accordance with the recommendations of [1].

To close the given system of equations we must compute the third moments $\langle v'_p v'_p v'_p\rangle$, $\langle v'_p w'_p w'_p\rangle$, $\langle v'_p v'_p w'_p\rangle$, and $\langle w'_p w'_p w'_p\rangle$ appearing in Eqs. (9) and (10). For this purpose we construct the equations of transfer of the correlations sought. We use the equations of transfer of the third moment $\langle w'_p w'_p w'_p\rangle$ to illustrate their derivation. Multiplying the pulsation equation (7) by $w'_p w'_p$, we obtain

$$\rho_p\beta\Big[u_p\frac{\partial w'^3_p}{3\partial z}+u'_p\frac{\partial w'^3_p}{3\partial z}+v_p\frac{\partial w'^3_p}{3\partial r}+v'_p\frac{\partial w'^3_p}{3\partial r}+\frac{1}{r}\Big(v_p w'^3_p+w'^3_p v'_p\Big)-$$

$$-w'^2_p\frac{\partial\langle u'_p w'_p\rangle}{\partial z}-\frac{w'^2_p\partial\left(r\langle w'_p v'_p\rangle\right)}{r\partial r}-\frac{w'^2_p\langle w'_p v'_p\rangle}{r}\Big]=F'_{a\varphi}w'^2_p\,. \tag{11}$$

We transform Eq. (11) using expressions (8) and the pulsation continuity equation premultiplied by $w'_p w'_p w'_p/3$. Thereafter we carry out averaging in the equation obtained. Disregarding the mixed third correlation moment $\langle w'_p w'_p w'_g\rangle$ in the approximation of a narrow channel on the portion of stabilized gas-suspension flow, we write the equation of transfer of the quantity $\langle w'_p w'_p w'_p\rangle$ sought:

$$\rho_p\beta\left[\frac{\partial\left(r\langle v'_p w'^3_p\rangle\right)}{3r\partial r}+\frac{\langle v'_p w'^3_p\rangle}{r}-\frac{\langle w'^2_p\rangle\langle w'_p v'_p\rangle}{r}-\frac{\langle w'^2_p\rangle\,\partial\left(r\langle w'_p v'_p\rangle\right)}{r\partial r}+\frac{\langle w'^3_p\rangle}{\tau}\right]=0\,. \tag{12}$$

We may obtain, in a similar manner, the following transfer equations for the remaining correlations sought:

$$\rho_p\beta\left[\frac{1}{3r}\frac{\partial}{\partial r}\Big(r\langle v'^4_p\rangle\Big)-\frac{\langle v'^2_p w'^2_p\rangle}{r}-\langle v'^2_p\rangle\frac{1}{r}\frac{\partial}{\partial r}\Big(r\langle v'^2_p\rangle\Big)+\frac{\langle v'^2_p\rangle\langle w'^2_p\rangle}{r}+\frac{\langle v'^3_p\rangle}{\tau}\right]=0\,, \tag{13}$$

$$\rho_p\beta\Bigg[\frac{\partial\left(r\langle v'^3_p w'_p\rangle\right)}{2r\partial r}-\frac{\langle w'^3_p v'_p\rangle}{r}+\frac{\langle w'_p v'_p\rangle\langle w'^2_p\rangle}{r}+\frac{\langle v'^3_p w'_p\rangle}{2r}-\frac{\langle v'^2_p\rangle\langle w'_p v'_p\rangle}{2r}-$$

$$-\frac{\langle v'^2_p\rangle\,\partial\left(r\langle w'_p v'_p\rangle\right)}{2r\partial r}-\frac{\langle w'_p v'_p\rangle\,\partial\left(r\langle v'^2_p\rangle\right)}{r\partial r}+\frac{3\langle v'^2_p w'_p\rangle}{2\tau}\Bigg]=0\,, \tag{14}$$

$$\rho_p\beta\Bigg[\frac{\partial\left(r\langle v'^2_p w'^2_p\rangle\right)}{2r\partial r}-\frac{\langle w'^4_p\rangle}{2r}+\frac{\langle w'^2_p\rangle^2}{2r}+\frac{\langle v'^2_p w'^2_p\rangle}{r}-\frac{\langle w'_p v'_p\rangle^2}{r}-$$

$$-\frac{\langle w'_p v'_p\rangle\,\partial\left(r\langle w'_p v'_p\rangle\right)}{r\partial r}-\frac{\langle w'^2_p\rangle\,\partial\left(r\langle v'^2_p\rangle\right)}{2r\partial r}+\frac{3\langle w'^2_p v'_p\rangle}{2\tau}\Bigg]=0\,. \tag{15}$$

To close Eq. (12) we must find the correlation of fourth order $\langle v'_p w'^3_p\rangle$ appearing in it. For this purpose we construct the equation of transfer of the correlation moment sought. We multiply the pulsation equation (6) by $w'^3_p/3$ and Eq. (11) by v'_p and combine the resulting equations:

$$\rho_p\beta\Bigg[u_p\frac{\partial v'_p w'^3_p}{3\partial z}+v_p\frac{\partial v'_p w'^3_p}{3\partial r}+u'_p\frac{\partial v'_p w'^3_p}{3\partial z}+v'_p\frac{\partial v'_p w'^3_p}{3\partial r}+\frac{v_p v'_p w'^3_p}{r}+$$

$$+\frac{v'^2_p w'^3_p}{r}-\frac{w'^2_p v'_p\langle w'_p v'_p\rangle}{r}+\frac{v'_p w'^3_p\partial v_p}{3\partial r}-\frac{w'^5_p}{3r}-\frac{w'^2_p v'_p\partial\langle u'_p w'_p\rangle}{\partial z}-\frac{w'^2_p v'_p\partial\left(r\langle w'_p v'_p\rangle\right)}{r\partial r}-$$

$$-\frac{w_p'^3 \partial \langle u_p' v_p' \rangle}{3\partial z} - \frac{w_p'^3 \partial (r \langle v_p'^2 \rangle)}{3r\partial r} + \frac{w_p'^3 \langle w_p'^2 \rangle}{3r} + \frac{u_p' w_p'^3 \partial v_p}{3\partial z}\Bigg] = F'_{a\varphi} w_p'^2 v_p' + \frac{F'_{ar} w_p'^3}{3} . \tag{16}$$

We transform Eq. (16) using expressions (8) and the pulsation continuity equation premultiplied by $v_p' w_p'^3/3$, after which carry out averaging in the equation transformed. Disregarding the mixed fourth correlation moments (gas–particle), in the approximation of a narrow channel on the portion of steady-state motion of the two-phase flow, we reduce the equation of transfer of the quantity $\langle v_p' w_p'^3 \rangle$ to the form

$$\rho_p \beta \Bigg[\frac{\partial (r \langle v_p'^2 w_p'^3 \rangle)}{3r\partial r} + \frac{\langle v_p'^2 w_p'^3 \rangle}{r} - \frac{\langle w_p'^2 v_p' \rangle \langle w_p' v_p' \rangle}{r} - \frac{\langle w_p'^5 \rangle}{3r} - \frac{\langle w_p'^2 v_p' \rangle \partial (r \langle w_p' v_p' \rangle)}{r\partial r} -$$

$$- \frac{\langle w_p'^3 \rangle \partial (r \langle v_p' \rangle)}{3r\partial r} + \frac{\langle w_p'^3 \rangle \langle w_p'^2 \rangle}{3r} \Bigg] = - \frac{4\rho_p \beta \langle v_p' w_p'^3 \rangle}{3\tau} . \tag{17}$$

Similarly to [5], the fifth correlation moments present in (17) may be represented as the sum of the products of the correlations of second and third orders. With allowance for this, Eq. (17) may be transformed to the form

$$\langle v_p' w_p'^3 \rangle = -\tau \Bigg[\frac{\langle v_p'^2 \rangle \partial \langle w_p'^3 \rangle}{4\partial r} + \frac{3 \langle w_p' v_p' \rangle \partial \langle w_p'^2 v_p' \rangle}{4\partial r} + \frac{3 \langle v_p'^2 \rangle \langle w_p'^3 \rangle}{4r} +$$

$$+ \frac{3 \langle w_p' v_p' \rangle \langle w_p'^2 v_p' \rangle}{2r} - \frac{3 \langle w_p'^2 \rangle \langle w_p'^3 \rangle}{4r} \Bigg]. \tag{18}$$

Substituting (18) into (12), we obtain the final form of the equation of transfer of the third correlation moment $\langle w_p'^3 \rangle$:

$$\beta \rho_p \Bigg[-\frac{\partial}{12r\partial r} \left(r\tau \langle v_p'^2 \rangle \frac{\partial \langle w_p'^3 \rangle}{\partial r} \right) - \frac{\partial}{4r\partial r} \left(r\tau \langle w_p' v_p' \rangle \frac{\partial \langle w_p'^2 v_p' \rangle}{\partial r} \right) - \frac{\partial (\tau \langle v_p'^2 \rangle \langle w_p'^3 \rangle)}{4r\partial r} -$$

$$- \frac{\partial (\tau \langle w_p' v_p' \rangle \langle w_p'^2 v_p' \rangle)}{2r\partial r} + \frac{\partial (\tau \langle w_p'^2 \rangle \langle w_p'^3 \rangle)}{4r\partial r} - \frac{\tau \langle v_p'^2 \rangle \partial \langle w_p'^3 \rangle}{4r\partial r} - \frac{3\tau \langle w_p' v_p' \rangle \partial \langle w_p'^2 v_p' \rangle}{4r\partial r} -$$

$$- \frac{3\tau \langle v_p'^2 \rangle \langle w_p'^3 \rangle}{4r^2} - \frac{3\tau \langle w_p' v_p' \rangle \langle w_p'^2 v_p' \rangle}{2r^2} + \frac{3\tau \langle w_p'^2 \rangle \langle w_p'^3 \rangle}{4r^2} - \frac{\langle w_p'^2 \rangle \langle w_p' v_p' \rangle}{r} -$$

$$- \frac{\langle w_p'^2 \rangle \partial (r \langle w_p' v_p' \rangle)}{r\partial r} + \frac{\langle w_p'^3 \rangle}{\tau} \Bigg] = 0 . \tag{19}$$

Equation (13) involves the fourth correlation moments, which may be expressed similarly to [5]:

$$\langle v_p' v_p' v_p' v_p' \rangle = 3 \langle v_p' v_p' \rangle \langle v_p' v_p' \rangle , \quad \langle v_p' v_p' w_p' w_p' \rangle = \langle v_p' v_p' \rangle \langle w_p' w_p' \rangle + 2 \langle w_p' v_p' \rangle \langle w_p' v_p' \rangle . \tag{20}$$

Substituting (20) into (13), after simple manipulations we have

$$\langle v_p'^3 \rangle = -\tau \left[\langle v_p'^2 \rangle \frac{\partial \langle v_p'^2 \rangle}{\partial r} - \frac{2 \langle w_p' v_p' \rangle^2}{r} \right]. \tag{21}$$

Equation (14) may be closed with a representation of the fourth moment $\langle v_p'^3 w_p' \rangle$ as the sum of the products of second moments:

$$\langle v_p'^3 w_p' \rangle = 3 \langle v_p'^2 \rangle \langle w_p' v_p' \rangle . \tag{22}$$

We transform Eq. (14) with account for expressions (18) and (22). Omitting cumbersome computations, we write the algebraic expression for the third moment $\langle v_p'^2 w_p' \rangle$:

$$\langle v_p'^2 w_p' \rangle = - \tau \left[\frac{2 \langle v_p'^2 \rangle \partial \langle w_p' v_p' \rangle}{3 \partial r} + \frac{\langle w_p' v_p' \rangle \partial \langle v_p'^2 \rangle}{3 \partial r} + \frac{2 \langle v_p'^2 \rangle \langle w_p' v_p' \rangle}{3r} + \right.$$

$$+ \frac{\tau \langle v_p'^2 \rangle \partial \langle w_p'^3 \rangle}{6 r \partial r} + \frac{\tau \langle w_p' v_p' \rangle \partial \langle w_p'^2 v_p' \rangle}{2 r \partial r} + \frac{\tau \langle v_p'^2 \rangle \partial \langle w_p'^3 \rangle}{2 r^2} + \frac{\tau \langle w_p' v_p' \rangle \partial \langle w_p'^2 v_p' \rangle}{r^2} -$$

$$\left. - \frac{\tau \langle w_p'^2 \rangle \langle w_p'^3 \rangle}{2 r^2} + \frac{2 \langle w_p' v_p' \rangle \langle w_p'^2 \rangle}{3r} \right] . \tag{23}$$

To close Eq. (15) we must determine the fourth moments $\langle v_p'^2 w_p'^2 \rangle$ and $\langle w_p'^4 \rangle$ appearing in it. Here we use a combined method for computation of the quantities sought. The value of $\langle v_p'^2 w_p'^2 \rangle$ is found on the basis of the Millionshchikov hypothesis assuming the equality of the cumulants of fourth order to zero and representing the fourth moments as the sum of the products of second moments. Finally we obtain

$$\langle v_p'^2 w_p'^2 \rangle = \langle v_p'^2 \rangle \langle w_p'^2 \rangle + 2 \langle w_p' v_p' \rangle^2 . \tag{24}$$

The correlation of fourth order $\langle w_p'^4 \rangle$ is calculated from the equation of its transfer in the approximation of a boundary layer on the portion of steady-state motion of a two-phase flow; this equation may be obtained similarly to Eq. (12):

$$\rho_p \beta \left[\frac{\partial (r \langle v_p' w_p'^4 \rangle)}{4 r \partial r} + \frac{\langle v_p' w_p'^4 \rangle}{r} - \frac{\langle w_p'^3 \rangle \langle w_p' v_p' \rangle}{r} - \frac{\langle w_p'^3 \rangle \partial (r \langle w_p' v_p' \rangle)}{r \partial r} + \frac{\langle w_p'^4 \rangle}{\tau} \right] = 0 . \tag{25}$$

Representing the fifth moment $\langle v_p' w_p'^4 \rangle$ in (25) as the sum of the products of second and third moments, we transform (25) to the form

$$\langle w_p'^4 \rangle = - \tau \left[\frac{\langle w_p' v_p' \rangle \partial \langle w_p'^3 \rangle}{\partial r} + \frac{3 \langle w_p' v_p' \rangle \langle w_p'^3 \rangle}{r} \right] . \tag{26}$$

Substituting (24) and (26) into (15), after simple manipulations we have

$$\langle w_p'^2 v_p' \rangle = - \tau \left[\frac{\langle v_p'^2 \rangle \partial \langle w_p'^2 \rangle}{3 \partial r} + \frac{2 \langle w_p' v_p' \rangle \partial \langle w_p' v_p' \rangle}{3 \partial r} + \frac{\tau \langle w_p' v_p' \rangle \partial \langle w_p'^3 \rangle}{3 r \partial r} + \right.$$

$$\left. + \frac{\tau \langle w_p' v_p' \rangle \langle w_p'^3 \rangle}{r^2} + \frac{\langle w_p'^2 \rangle^2}{3r} + \frac{2 \langle v_p'^2 \rangle \langle w_p'^2 \rangle}{3r} + \frac{2 \langle w_p' v_p' \rangle^2}{3r} \right] . \tag{27}$$

We transform Eqs. (9) and (10) using expressions (21), (23), and (27). As a result, the equation of transfer of the quantity $\langle w_p' v_p' \rangle$ will be represented in the form

$$\beta \rho_p \left[\frac{2 \partial}{3 r \partial r} \left(r \tau \langle v_p'^2 \rangle \frac{\partial \langle w_p' v_p' \rangle}{\partial r} \right) + \frac{\partial}{3 r \partial r} \left(r \tau \langle w_p' v_p' \rangle \frac{\partial \langle v_p'^2 \rangle}{\partial r} \right) + \frac{2 \partial (\tau \langle v_p'^2 \rangle \langle w_p' v_p' \rangle}{3 r \partial r} + \right.$$

$$+\frac{\partial}{6r\partial r}\left(\tau^2\langle v_p'^2\rangle\frac{\partial\langle w_p'^3\rangle}{\partial r}\right)+\frac{\partial}{2r\partial r}\left(\tau^2\langle w_p'v_p'\rangle\frac{\partial\langle w_p'^2v_p'\rangle}{\partial r}\right)+\frac{\partial}{2r\partial r}\left(\frac{\tau^2\langle v_p'^2\rangle\langle w_p'^3\rangle}{r}\right)+$$

$$+\frac{\partial}{r\partial r}\left(\frac{\tau^2\langle w_p'v_p'\rangle\langle w_p'^2v_p'\rangle}{r}\right)-\frac{\partial}{2r\partial r}\left(\frac{\tau^2\langle w_p'^2\rangle\langle w_p'^3\rangle}{r}\right)+\frac{2\partial(\tau\langle w_p'v_p'\rangle\langle w_p'^2\rangle)}{3r\partial r}+\frac{\langle w_p'^3\rangle}{r}+$$

$$+\frac{2\tau\langle v_p'^2\rangle\partial\langle w_p'v_p'\rangle}{3r\partial r}+\frac{\tau\langle w_p'v_p'\rangle\partial\langle v_p'^2\rangle}{3r\partial r}+\frac{2\tau\langle v_p'^2\rangle\langle w_p'v_p'\rangle}{3r^2}+\frac{\tau^2\langle v_p'^2\rangle\partial\langle w_p'^3\rangle}{6r^2\partial r}+$$

$$+\frac{\tau^2\langle w_p'v_p'\rangle\partial\langle w_p'^2v_p'\rangle}{2r^2\partial r}+\frac{\tau^2\langle v_p'^2\rangle\langle w_p'^3\rangle}{2r^3}+\frac{\tau^2\langle w_p'v_p'\rangle\langle w_p'^2v_p'\rangle}{r^3}-\frac{\tau^2\langle w_p'^2\rangle\langle w_p'^3\rangle}{2r^3}+$$

$$\left.+\frac{2\tau\langle w_p'v_p'\rangle\langle w_p'^2\rangle}{3r^2}\right]+\frac{\rho_p\beta}{\tau}\left(\langle v_g'w_p'\rangle+\langle v_p'w_g'\rangle-2\langle w_p'v_p'\rangle\right)=0\,. \tag{28}$$

The equation of transfer of the quantity $\langle v_p'v_p'\rangle$ is

$$\rho_p\beta\left[\frac{\partial}{r\partial r}\left(r\tau\langle v_p'^2\rangle\frac{\partial\langle v_p'^2\rangle}{\partial r}\right)-\frac{2}{r}\frac{\partial(\tau\langle w_p'v_p'\rangle^2)}{\partial r}-\frac{2\tau\langle v_p'^2\rangle}{3r}\frac{\partial\langle w_p'^2\rangle}{\partial r}-\right.$$

$$-\frac{4\tau\langle w_p'v_p'\rangle}{3r}\frac{\partial\langle w_p'v_p'\rangle}{\partial r}-\frac{2\tau^2\langle w_p'v_p'\rangle\partial\langle w_p'^3\rangle}{3r^2\partial r}-\frac{2\tau^2\langle w_p'v_p'\rangle\langle w_p'^3\rangle}{r^3}-\frac{2\tau\langle w_p'^2\rangle^2}{3r^2}-$$

$$\left.-\frac{4\tau\langle v_p'^2\rangle\langle w_p'^2\rangle}{3r^2}-\frac{4\tau\langle w_p'v_p'\rangle^2}{3r^2}\right]+\frac{2\rho_p\beta}{\tau}\left(\langle v_g'v_p'\rangle-\langle v_p'^2\rangle\right)+$$

$$+2\left\{\frac{\delta^2\rho_p}{6912\beta}\left(\frac{\partial u_p}{\partial r}\right)^2\left(\frac{1-K_n}{2}-\frac{1-K_\tau}{7}\right)^2-C_1\rho_p\beta\langle v_p'v_p'\rangle\left(1-K_n^2\right)\right\}N=0\,. \tag{29}$$

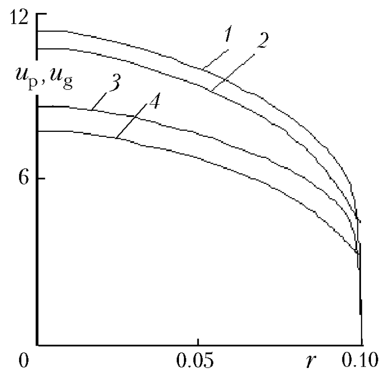
The equation of transfer of the quantity $\langle w_p'w_p'\rangle$ is

$$\rho_p\beta\left[\frac{\partial}{3r\partial r}\left(\frac{r\tau\langle v_p'^2\rangle\partial\langle w_p'^2\rangle}{\partial r}\right)+\frac{2\partial}{3r\partial r}\left(\frac{r\tau\langle w_p'v_p'\rangle\partial\langle w_p'v_p'\rangle}{\partial r}\right)+\frac{\partial}{3r\partial r}\left(\frac{\tau^2\langle w_p'v_p'\rangle\partial\langle w_p'^3\rangle}{\partial r}\right)+\right.$$

$$+\frac{\partial}{r\partial r}\left(\frac{\tau^2\langle w_p'v_p'\rangle\langle w_p'^3\rangle}{r}\right)+\frac{\partial(\tau\langle w_p'^2\rangle^2)}{3r\partial r}+\frac{2\partial(\tau\langle v_p'^2\rangle\langle w_p'^2\rangle)}{3r\partial r}+\frac{2\partial(\tau\langle w_p'v_p'\rangle^2)}{3r\partial r}+$$

$$+\frac{2\tau\langle v_p'^2\rangle\partial\langle w_p'^2\rangle}{3r\partial r}+\frac{4\tau\langle w_p'v_p'\rangle\partial\langle w_p'v_p'\rangle}{3r\partial r}+\frac{2\tau^2\langle w_p'v_p'\rangle\partial\langle w_p'^3\rangle}{3r^2\partial r}+\frac{2\tau^2\langle w_p'v_p'\rangle\langle w_p'^3\rangle}{r^3}+$$

$$\left.+\frac{2\tau\langle w_p'^2\rangle^2}{3r^2}+\frac{4\tau\langle v_p'^2\rangle\langle w_p'^2\rangle}{3r^2}+\frac{4\tau\langle w_p'v_p'\rangle^2}{3r^2}\right]+\frac{2\rho_p\beta}{\tau}\left(\langle w_g'w_p'\rangle-\langle w_p'^2\rangle\right)+$$



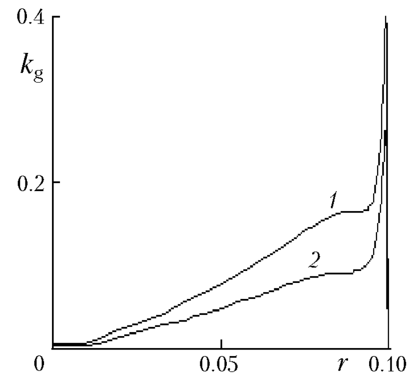


Fig. 1. Profiles of the averaged axial velocities of the gas and particles: variant I, 1) u_g and 2) u_p; variant II, 3) u_g and 4) u_p.

Fig. 2. Profiles of the kinetic energies of turbulent gas-velocity pulsations k: 1) variant I; 2) variant II.

$$+2\left\{\frac{\delta^2\rho_p}{6912\beta}\left(\frac{\partial u_p}{\partial r}\right)^2\left(\frac{1-K_n}{2}-\frac{1-K_\tau}{7}\right)^2-C_2\rho_p\beta\,\langle w_p'w_p'\rangle\left(1-K_n^2\right)\right\}N=0\,. \tag{30}$$

Boundary conditions on the flow axis for Eqs. (1), (2), (19), and (28)–(30) are specified from symmetry considerations:

$$\begin{gathered}\left(\partial u_g/\partial r\right)_{ax}=\left(\partial k_g/\partial r\right)_{ax}=\left(\partial\,\langle w_p'w_p'w_p'\rangle/\partial r\right)_{ax}=0\,,\\ \left(\partial u_p/\partial r\right)_{ax}=\left(\partial\,\langle v_p'v_p'\rangle/\partial r\right)_{ax}=\left(\partial\,\langle w_p'w_p'\rangle/\partial r\right)_{ax}=\left(\partial\,\langle w_p'v_p'\rangle/\partial r\right)_{ax}=0\,;\end{gathered} \tag{31}$$

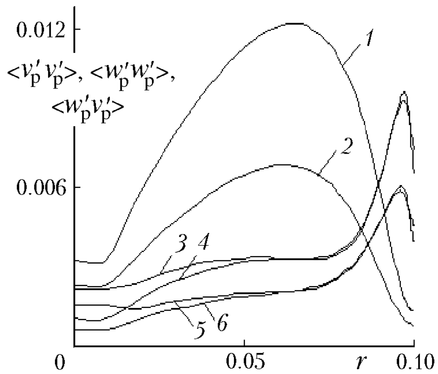
on the channel wall, they are specified by the relations

$$u_{g,w}=k_{g,w}=0\,,\quad u_{p,w}=\frac{\delta}{24\sqrt{2}\,\beta\,(1-K_\tau)}\left(\frac{\partial u_p}{\partial r}\right)_w(7K_n-2K_\tau-5)\,, \tag{32}$$

$$\left(\partial\,\langle v_p'v_p'\rangle/\partial r\right)_w=\left(\partial\,\langle w_p'w_p'\rangle/\partial r\right)_w=\left(\partial\,\langle w_p'v_p'\rangle/\partial r\right)_w=\left(\partial\,\langle w_p'w_p'w_p'\rangle/\partial r\right)_w=0\,. \tag{33}$$

The given system of equations (1), (2), (19), and (28)–(30) with boundary conditions (31)–(33) has been integrated by the methods of direct and reverse marchings on a nonuniform grid clustering at the channel wall; the pressure gradient was eliminated using the well-known method [6]. In accordance with the above algorithm, we developed a program for calculation of two-phase flows.

Let us discuss results of calculations of two variants for the following initial data: β = 0.0012, ρ_g = 1.3 kg/m^3, and ρ_p = 1600 kg/m^3 (Variant I: $\delta = 0.14\cdot10^{-3}$ m and $u_{g,m}$ = 8.5 m/sec; variant II: $\delta = 0.18\cdot10^{-3}$ m and $u_{g,m}$ = 6.5 m/sec.) Figures 1–5 give certain results of calculating the aerodynamics of a gasdispersed flow in a channel of radius R = 0.1 m. Figure 1 illustrates the distribution of the averaged axial velocities of the gas particles on the portion of steady-state motion. In the flow core, the dispersed phase lags behind the gas the larger, the higher the free-fall velocity of the particles. In the wall region where the velocity of the carrier phase sharply decreases, particles lead the gas: here the drag force is negative, and the suspension of the particles is caused by the action of Reynolds stresses (Fig. 1, curves 2 and 4). Increase in the average (over the cross section) velocity of the carrier medium $u_{g,m}$ contributes to the generation of the turbulent gas energy, because of which the kinetic pulsation energy of the gas in variant I turns out to be higher than that in variant II (Fig. 2, curves 1 and 2).



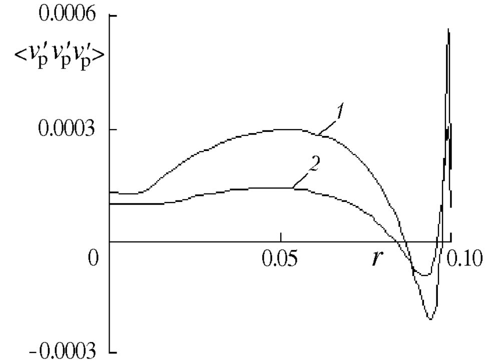


Fig. 3. Profiles of the second correlation moments of dispersed-phase-velocity pulsations: variant I, 1) $\langle w'_p v'_p \rangle$, 3) $\langle w'_p w'_p \rangle$, and 4) $\langle v'_p v'_p \rangle$; variant II, 2) $\langle w'_p v'_p \rangle$, 5) $\langle w'_p w'_p \rangle$, and 6) $\langle v'_p v'_p \rangle$.

Fig. 4. Distribution of the third correlation moment $\langle v'_p v'_p v'_p \rangle$ over the flow cross section: 1) variant I; 2) variant II.

Figure 3 gives the profiles of the second correlation moments of dispersed-phase-velocity pulsations. As the calculation results show, the behavior of the dependence $\langle v'_p v'_p \rangle(r)$ (curve 4) is mainly determined by the rate of generation of the energy of random (turbulent and pseudoturbulent) particle motion; on the one hand, it is caused by the action of the drag force (tenth term of Eq. (29), $2\rho_p\beta\langle v'_p v'_g \rangle/\tau$), on the other, by interparticle collisions (twelfth term of Eq. (29) $\dfrac{2\delta^2\rho_p(du_p/dr)^2 N}{6912\beta}\left(\dfrac{1-K_n}{2}-\dfrac{1-K_\tau}{7}\right)^2$). On the ascending branch $0.0095 < r < 0.059$ m, we have a monotonic increase in the function $\langle v'_p v'_p \rangle(r)$ due to the increase in the production of the energy of random particle motion; it is caused by the increase in the mixed correlation moment $\langle v'_p v'_g \rangle$ and the modulus of the gradient of axial velocity of the particles $|\partial u_p/\partial r|$ (Fig. 1, curve 2). The rate of generation of the turbulent dispersed-phase energy decreases on the portion $0.059 < r < 0.078$ m, which finally leads to a reduction in the derivative $\partial\langle v'_p v'_p \rangle/dr$. In the interval $0.078 < r < 0.098$ m, the rate of generation of the pseudoturbulent energy sharply increases due to the significant increase in the derivative $|\partial u_p/\partial r|$. This ensures a rapid growth in the function $\langle v'_p v'_p \rangle(r)$ in this zone. The rate of production of the energy of random particle motion noticeably decreases in the wall region $r > 0.098$ m, which contributes to the decrease in the $\langle v'_p v'_p \rangle(r)$ curve.

It is seen in Fig. 3 that the $\langle w'_p w'_p \rangle(r)$ curves are similar to the $\langle v'_p v'_p \rangle(r)$ curves; the values of the functions $\langle w'_p w'_p \rangle(r)$ and $\langle v'_p v'_p \rangle(r)$ are virtually coincident in the peripheral region, whereas in the axial zone, they somewhat differ from each other, which suggests the anisotropy of the field of pulsation energy of the solid phase in this zone (Fig. 3; curves 3 and 4 and 5 and 6 are compared).

Figure 4 gives results of calculating the third moment of pulsations of the radial particle velocity $\langle v'_p v'_p v'_p \rangle$ on the portion of steady-state motion of a two-phase flow. In the interval $0.0095 < r < 0.055$ m, the second term of the right-hand side of Eq. (21) has a dominant role in the formation of the $\langle v'_p v'_p v'_p \rangle(r)$ profile (curve 1). The monotonic increase in the dependence $\langle v'_p v'_p v'_p \rangle(r)$ is related to the rapid growth in the $\langle w'_p v'_p \rangle(r)$ curve in the range in question (Fig. 3, curve 1). On the portion $0.055 < r < 0.095$ m, we have an increase in the first term of the right-hand side of the equation indicated due to the growth in the $\langle v'_p v'_p \rangle(r)$ curve and its derivative (Fig. 3, curve 4). The value of the second term decreases, which is caused by the decrease in the dependence $\langle w'_p v'_p \rangle(r)$ and by the growth in the coordinate r. The values of both terms of Eq. (21) become equal at the point $r = 0.0868$ m but opposite in sign; therefore, the function $\langle v'_p v'_p v'_p \rangle(0.0868)$ is equal to zero. In the range $0.0868 < r < 0.095$ m, where the dependence $\langle v'_p v'_p \rangle(r)$ sharply grows, the first term begins to prevail over the second term, which ensures further decrease in the $\langle v'_p v'_p v'_p \rangle(r)$ curve in this region. In the interval $0.095 < r < 0.0972$ m, the first term decreases to the value of the second term ($\partial\langle v'_p v'_p \rangle/dr \to 0$) and, as a consequence, the function $\langle v'_p v'_p v'_p \rangle(r)$ tends to zero. The dependence $\langle v'_p v'_p v'_p \rangle(r)$ continues

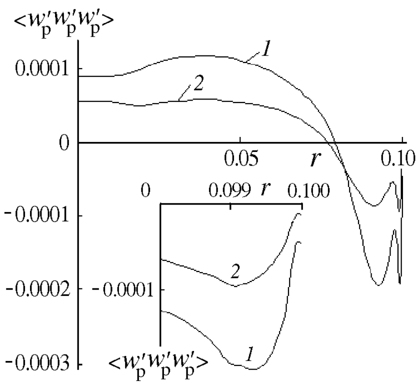


Fig. 5. Distribution of the correlation of third order of particle-velocity pulsations $\langle w_p'^3 \rangle$ on the portion of steady-state motion of a two-phase flow: 1) variant I; 2) variant II.

to grow in the wall range $0.0972 < r < 0.0994$ m, which is related to the change of sign in the derivative $\partial\langle v_p' v_p' \rangle / dr$ and to the increase in its absolute value.

Figure 5 gives the values of the correlation of third order of pulsations of the transverse particle velocity $\langle w_p'^3 \rangle$ on the portion of stabilized gas-suspension flow. It is seen in the figure that the function $\langle w_p'^3 \rangle(r)$ has its maximum at the point $r = 0.045$ m (curve 1); the maximum is attributable to the influence of the eleventh and twelfth terms of Eq. (19). The monotonic increase in the $\langle w_p'^3 \rangle(r)$ curve on the ascending branch $0.018 < r < 0.045$ m is caused by the growth in the dependences $\langle w_p' w_p' \rangle(r)$ and $\langle w_p' v_p' \rangle(r)$ in this region (Fig. 3, curves 1 and 3). The decrease in the function $\langle w_p'^3 \rangle(r)$ on the portion $0.045 < r < 0.064$ m is related to the significant decrease in the twelfth term of the equation indicated ($\partial\langle w_p' v_p' \rangle / dr \to 0$). In the interval $0.064 < r < 0.08$ m, the derivative $\partial\langle w_p' v_p' \rangle / dr$ changes its sign, and its value (in modulus) grows. The moduli of the eleventh and twelfth terms of the equation become equal at the point $r = 0.08$ m; as a consequence, $\langle w_p'^3 \rangle(0.08)$ is equal to zero. As the coordinate r grows further ($0.08 < r < 0.094$ m), the influence of the twelfth term of the equation turns out to be much higher than that of the eleventh term, which is caused by the significant growth in the value of $|\partial\langle w_p' v_p' \rangle / dr|$. Therefore, the dependence $\langle w_p'^3 \rangle(r)$ continues to decrease in this zone. In the wall region $0.094 < r < 0.098$ m, the behavior of the $\langle w_p'^3 \rangle(r)$ curve begins to be affected by the second term of the equation; as a result, the function $\langle w_p'^3 \rangle(r)$ grows on this portion.

Thus, the mathematical model proposed makes it possible to obtain detailed information on the averaged and pulsation characteristics of the carrier flow and the dispersed phase and may be useful in designing technical devices intended for pneumatic transport of bulk materials, removal of solid impurities from gases, mechanical and thermal treatment of powders, and burning of solid fuel.

NOTATION

C_1, C_2, and σ, empirical constants; F, force, kg/(sec^2·m^2); G, generation of the turbulent gas energy in the trails behind particles, kg/(sec^3·m); g, free-fall acceleration, m/sec^2; K, recovery factor of velocity on impact; k, kinetic pulsation energy, m^2/sec^2; N, impact frequency, 1/sec; P, gas pressure, N/m^2; R, channel radius, m; r, z, φ, radial, longitudinal, and transverse coordinates, m; u, v, and w, averaged components of the velocity vector, m/sec; β, true volume concentration of particles; δ, particle diameter, m; ε, pulsation-energy dissipation, m^2/sec^3; η, kinematic viscosity, m^2/sec; ρ, density, kg/m^3; τ, dynamic-relaxation time, sec. Subscripts and superscripts: a, aerodynamic; g, gas; m, mean (average) over the cross section; n, normal; ax, flow axis; p, particle; t, turbulent; w, wall; τ, tangential; ′, pulsation component in time averaging; $\langle\ \rangle$, time averaging.

REFERENCES

1. A. A. Shraiber, L. B. Gavin, V. A. Naumov, and V. P. Yatsenko, *Turbulent Gas-Suspension Flows* [in Russian], Naukova Dumka, Kiev (1987).

2. L. V. Kondrat'ev, *A Model and Numerical Investigation of a Turbulent Gas-Suspension Flow in a Tube*, Author's Abstract of Candidate's Dissertation (Physics and Mathematics), Leningrad (1989).
3. B. B. Rokhman and A. A. Shraiber, Mathematical modeling of aerodynamics and physicochemical processes in the freeboard zone of a circulating fluidized bed furnace. 1. Statement of the problem. Basic aerodynamic equations, *Inzh.-Fiz. Zh.*, **65**, No. 5, 521–526 (1993).
4. B. B. Rokhman and A. A. Shraiber, Mathematical modeling of aerodynamics and physicochemical processes in the freeboard zone of a circulating fluidized bed furnace. 2. Interaction of particles (pseudoturbulence), *Inzh.-Fiz. Zh.*, **66**, No. 2, 159–167 (1994).
5. K. Hanjalic and B. E. Launder, A Reynolds stress model of turbulence and its application to thin shear flows, *J. Fluid. Mech.*, **52**, No. 4, 609–638 (1972).
6. L. M. Simuni, Numerical solution for nonisothermal motion of a viscous liquid in a two-dimensional tube, *Inzh.-Fiz. Zh.*, **10**, No. 1, 86–91 (1966).